

[illegible]

Question Paper Code : 30069

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2026.

Second Semester

Civil Engineering

MA25C02 — LINEAR ALGEBRA

(Common to All Branches (Except Electrical and Electronics Engineering/
Electronics and Instrumentation Engineering/Instrumentation and Control
Engineering/Electrical and Computer Engineering))

(Regulations 2025)

Time : 3 hours

Maximum : 100 marks

CO – Course Outcomes

CO1	Explain the fundamental concepts of Linear Algebra.
CO2	Compute and interpret eigenvalues and eigenvectors.
CO3	Apply inner product concepts and perform orthogonalization.
CO4	Compute least squares solutions of linear system of equations.
CO5	Use MATLAB to implement and validate key linear algebra concepts.

BL – Bloom's Taxonomy Levels

(L1-Remember, L2-Understand, L3-Apply, L4-Analyze, L5-Evaluate, L6-Create)

Answer ALL questions.

PART – A (Total Marks = 40)

[Q. No. 1 to Q. No. 10 ; $10 \times 1 = 10$ Marks (Multiple Choice Questions)]

Q. No. 11 to Q. No. 20; $10 \times 3 = 30$ Marks]

Q.No.	Questions	Marks	CO	BL
1.	Which of the following spans R^2 ? (A) $\{(1, 2), (2, 4)\}$ (B) $\{(1, 0), (0, 1)\}$ (C) $\{(1, 1)\}$ (D) $\{(0, 0), (1, 1)\}$	1	1	1

2.	Dimension of the subspace $\{(x, y, z) \in \mathbb{R}^3 : x - 2y + z = 0\}$ is (A) 1 (B) 2 (C) 3 (D) 0	1	1	2
3.	By dimension theorem : $\text{rank}(T) + \text{nullity}(T) =$ (A) $\dim(\text{domain})$ (B) $\dim(\text{codomain})$ (C) $\dim(\text{range})$ (D) $\dim(\text{kernel})$	1	2	1
4.	A matrix A is diagonalizable if (A) A has n eigenvalues (B) A has n linearly dependent eigenvectors (C) A has n linearly independent eigenvectors (D) $\det(A) \neq 0$	1	2	2
5.	The norm of $(1, -2, 2)$ is (A) 1 (B) 5 (C) 9 (D) 3	1	3	1
6.	If the inner product $\langle u, v \rangle = 0$, vectors are (A) parallel (B) orthogonal (C) linearly dependent (D) unit vectors	1	3	2
7.	A symmetric matrix is positive definite if (A) All eigenvalues > 0 (B) Determinant > 0 (C) Trace > 0 (D) All entries > 0	1	4	1
8.	QR decomposition of A gives (A) Q orthogonal, R upper triangular (B) Q lower triangular, R orthogonal (C) Q upper triangular, R orthogonal (D) Q lower triangular, R upper triangular	1	4	2

9.	To compute rank in MATLAB (A) trace(A) (B) det(A) (C) null(A) (D) rank(A)	1	5	1
10.	$[V, D] = \text{eig}(A)$ returns (A) V = eigenvectors, D = eigenvalues diagonal (B) V = eigenvalues, D = eigenvectors (C) V = singular vectors, D = eigenvalues (D) V = eigenvalues, D = singular vectors	1	5	2
11.	Check whether $S = \{(1, 2, 3), (0, 1, 2), (2, 3, 4)\}$ is linearly independent in R^3 .	3	1	2
12.	Find a basis for the subspace $W = \{(x, y, z) : x + y + z = 0\}$ in R^3 .	3	1	2
13.	Find the matrix of $T: R^2 \rightarrow R^2$, $T(x, y) = (3x - y, x + 2y)$ with respect to the standard basis.	3	2	2
14.	Find eigenvalues and eigenvectors of $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.	3	2	2
15.	Find the angle between $u = (1, -1)$ and $v = (2, 3)$	3	3	2
16.	Verify Cauchy-Schwarz inequality for $u = (2, -1)$, $v = (1, 2)$	3	3	1
17.	Find least squares solution to $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$.	3	4	2
18.	Orthogonally diagonalize the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.	3	4	1
19.	Write any one software code to check if vectors $[1, 2, 3]$, $[4, 5, 6]$, $[7, 8, 9]$ are linearly independent.	3	5	2
20.	Apply Gram-Schmidt to $\{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ using any software.	3	5	2

PART – B (5 × 12 = 60 marks)

Q.No.	Questions	Marks	CO	BL
21.	(a) Determine whether the set of all vectors of the form (x, y) in R^2 with standard operation of addition and scalar multiplication $k(x, y) = (kx, 0)$ is a vector space. If not, list the axioms that fail to hold.	12	1	4
	OR			
	(b) Find a basis and the dimension of solution space of the homogeneous system $x_1 + 3x_2 - 2x_3 + 2x_5 = 0$; $2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 = 0$; $5x_3 + 10x_4 + 15x_6 = 0$; $2x_1 + 6x_2 + 8x_4 + 4x_5 + 18x_6 = 0$.	12	1	4
22.	(a) Find bases for the eigenspaces of $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$	12	2	4
	OR			
	(b) Diagonalize the following matrix, if possible $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$	12	2	3
23.	(a) (i) Let $u = (a, b)$ and $v = (c, d)$ be vectors in R^2 . Verify that the weighted Euclidean inner product $\langle u, v \rangle = 3ac + 2bd$ satisfies the four inner product axioms.	8	3	3
	(ii) Prove that the following identity holds for vectors in any inner product space $\langle u, v \rangle = \frac{1}{4} \ u + v\ ^2 - \frac{1}{4} \ u - v\ ^2$.	4		
	OR			

	(b)	Assume that the vector space R^3 has the Euclidean inner product. Apply the Gram-Schmidt process to transform the basis vectors $u_1 = (1, 1, 1)$, $u_2 = (0, 1, 1)$, $u_3 = (0, 0, 1)$ into an orthogonal basis $\{v_1, v_2, v_3\}$, and then normalize the orthogonal basis vectors to obtain an orthonormal basis $\{q_1, q_2, q_3\}$.	12	3	4
24.	(a)	Find a QR-decomposition of $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}.$	12	4	3
	OR				
	(b)	Find the least squares solution, the least squares error vector, and the least squares error of the linear system $\begin{aligned} x_1 - x_2 &= 4 \\ 3x_1 + 2x_2 &= 1 \\ -2x_1 + 4x_2 &= 3 \end{aligned}$	12	4	3
25.	(a)	Using any software (i) Define the vectors $v_1 = (2, 1)$, $v_2 = (1, 2)$ (ii) Implement the Gram-Schmidt process to transform $\{v_1, v_2\}$ into an orthogonal set of vectors $\{u_1, u_2\}$. (iii) Normalize the orthogonal vectors to create an orthonormal set $\{q_1, q_2\}$.	12	5	5
	OR				

	(b)	In any software	12	5	5
	(i)	Create the symmetric matrix $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$.			
	(ii)	Use the <code>eig</code> function to compute the eigenvalues and eigenvectors of A .			
	(iii)	Construct the matrix P whose columns are the eigenvectors, and the diagonal matrix D whose diagonal entries are the eigenvalues.			
	(iv)	Verify that $A = PDP^{-1}$ by computing the right-hand side and comparing it to A .			